EXERCISES [MAI 1.4-1.5]

ARITHMETIC SEQUENCES

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

SEQUENCES (IN GENERAL)

1. (a) $u_1 = 2$, $S_1 = 2$.

(b) $u_1 = 5$, $S_2 = 7$.

(c) $S_5 = 2 + 5 + 10 + 3 + 7 = 27$.

2. (a) $u_2 = 2$, $u_{20} = 20$.

(b) $S_5 = 1 + 2 + 3 + 4 + 5 = 15$.

(c) $u_n = n$.

3. (a) 10, 20, 30

(b) $u_{10} = 100$

(c) $S_1 = 10$, $S_2 = 30$, $S_3 = 60$

4. (a) 10, 40, 90

(b) $u_{10} = 1000$

(c) $S_1 = 10$, $S_2 = 40$, $S_3 = 100$

5. (a) 10, 100, 1000

(b) $u_{10} = 10{,}000{,}000{,}000$

(c) $S_1 = 10$, $S_2 = 110$, $S_3 = 1110$

6. (a) 10, 20, 30

(b) $S_1 = 10$, $S_2 = 30$, $S_3 = 60$

7. (a) 10, 30, 70

(b) $S_1 = 10$, $S_2 = 40$, $S_3 = 110$

8. (a) $\sum_{r=1}^{3}(2r) = 2 + 4 + 6 = 12$

(b) $\sum_{r=1}^{3} r^2 = 1 + 4 + 9 = 14$

(c) $\sum_{r=1}^{3} 2^r = 2 + 4 + 8 = 14$

9. (a) $A = \sum_{r=1}^{10}(2r^2 + 1) = 780$, $B = \sum_{r=1}^{20}(2r^2 + 1) = 5760$,

$C = \sum_{r=11}^{20}(2r^2 + 1) = 4980$

(b) $A + C = B$ or $C = B - A$

ARITHMETIC SEQUENCES

10. (a) $u_1 = 11$, $d = 4$.

(b) $u_{101} = 11 + 100 \times 4 = 411$, $S_{101} = \frac{101}{2}(11 + 411) = 21311$

(c) $S_{20} = \frac{20}{2}(2 \times 11 + 19 \times 4) = 980$

(d) $u_n = 11 + (n-1)4 = 11 + 4n - 4 = 4n + 7$

(e) $u_n = 51 \Leftrightarrow 4n + 7 = 51 \Leftrightarrow 4n = 44 \Leftrightarrow n = 11$

11. (a) 3, 6, 9

(b) (i) **METHOD 1** By GDC $\sum_{n=1}^{20} 3n = 630$

METHOD 2 $S_{20} = \frac{20}{2}\ 2\times 3 + (20-1)\times 3 = 630$

(ii) **METHOD 1** By GDC $\sum_{n=21}^{100} 3n = 14520$

METHOD 2 AS with $u_1 = 3$, $d = 3$ $S_{100} - S_{20} = 15150 - 630 = 14520$

METHOD 3 AS with $u_1 = 63$, $d = 3$ $S_{80} = 14520$

12. (a) $u_1 = 1, u_2 = –1, u_3 = –3$

(b) By GDC $\sum_{n=1}^{20}(3-2n) = –360$ or by formula $S_{20} = \frac{20}{2}(2 \times 1 + 19 \times –2) = –360$

13. (a) $u_1 = 7, d = 2.5$
$u_{41} = u_1 + (n – 1)d = 7 + (41 – 1)2.5 = 107$

(b) $S_{101} = \frac{n}{2}[2u_1 + (n – 1)d] = \frac{101}{2}[2(7) + (101 – 1)2.52] = \frac{101(264)}{2} = 13332$

14. $S_5 = \frac{5}{2}\{2 + 32\} = 85$

15. (a) $8 = 2 + 2d \Rightarrow d = 3$
(b) $u_{20} = 2 + 19\times 3 = 59$

(c) $S_{20} = \frac{20}{2}(2 + 59) = 610$ or $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3) = 610$

16. $u_4 = 40 \Rightarrow u_1 + 3d = 40 \Rightarrow d = \frac{35}{3}$

$u_2 = 5 + \frac{35}{3} = 16\frac{2}{3}$ or $\frac{50}{3}$ or 16.7 (3 s.f.)

17. (a) $d = 3$
$u_{101} = 2 + 100 \times 3 = 302$

(b) $u_n = 152 \Leftrightarrow 152 = 2 + (n – 1) \times 3 \Leftrightarrow n = 51$

18. (a) $d = 6$

(b) $u_n = 1353 \Leftrightarrow 1353 = 3 + (n – 1)6 \Leftrightarrow n = 226$

(c) $S_{226} = \frac{226(3+1353)}{2} = 153\ 228$ (accept 153 000)

19. (a) $d = 2 \quad (u_1 = 7)$

(b) (i) $5 + 2n = 115 \Leftrightarrow n = 55$

(ii) $S_{55} = \dfrac{55}{2}(7+115) = 3355$ **OR**

$S_{55} = \dfrac{55}{2}(2(7)+54(2)), = 3355$ **OR**

$\sum_{k=1}^{55}(5+2k) = 3355$

20. (a) $u_{27} = u_1 + 26\text{ d} \Leftrightarrow 263 = u_1 + 26 \times 11 \Leftrightarrow u_1 = -23$

(b) (i) $516 = -23 + (n-1) \times 11 \Leftrightarrow n = 50$

(ii) $S_{50} = \dfrac{50(-23+516)}{2} = 12325$ **OR**

$S_{50} = \dfrac{50(2\times(-23)+49\times 11)}{2} = 12325$ (accept 12300)

21. $u_n = 417 \Leftrightarrow 17 + (n-1)10 = 417 \Leftrightarrow n = 41$ **OR** directly n = 41 (by observation)

$S_{41} = \dfrac{41}{2}(2(17) + 40(10)) = 8897$ **OR**

$S_{41} = \dfrac{41}{2}(17 + 417) = \dfrac{41}{2}(434) = 8897$

22. (a) $n = 51$, $S_{51} = 4182$

(b) $\sum_{r=1}^{51}(3r+4) = 4182$ (it is same sum!)

23. (a) $S_n = \dfrac{n}{2}(2 \times 2 + 3(n-1)) = \dfrac{n}{2}(3n + 1)$

(b) $\dfrac{n}{2}(3n + 1) = 1365 \Rightarrow 3n^2 + n - 2730 = 0 \Rightarrow n = 30$ or $n = \dfrac{-91}{3}$, hence $n = 30$

24. (a) $u_1 = S_1 = 7$

(b) $u_2 = S_2 - u_1 = 18 - 7 = 11$

$d = 11 - 7 = 4$

(c) $u_4 = u_1 + 3d = 7 + 3\times 4 = 19$ (OR directly 7, 11, 15, 19 so $u_4 = 19$)

25. (a) $d = 8$ (b) $u_{11} = 75$ (c) $S_{11} = 385$ (d) $n = 21$

26. (a) $u_4 = u_1 + 3d$ or $16 = -2 + 3d \Leftrightarrow d = 6$

(b) $u_n = u_1 + (n-1)\text{d} \Leftrightarrow 11998 = -2 + (n-1)6 \Leftrightarrow n = \dfrac{11998+2}{6} + 1 = 2001$

27. (a) $u_{20} = u_1 + 19d \Leftrightarrow 64 = 7 + 19d, \Leftrightarrow d = 3$

(b) $3709 = 7 + 3(n-1) \Leftrightarrow 3709 = 3n + 4 \Leftrightarrow n = 1235$

28. (a) $u_1 + 4d = 30 \quad u_1 + 12d = 70$

$u_1 = 10 \quad d = 5$

(b) $u_n = 5n+5$

(c) $S_n = \frac{n}{2}(5n+15)$

(d) $u_{20} = 105$ and $S_{20} = 1150$

29. (a) (i) $-37 = u_1 + 20d$ and $-3 = u_1 + 3d \Leftrightarrow d = -2 \quad u_1 = 3$

(b) $S_{10} = \frac{10}{2}(2\times 3 + 9(-2)) = -60$

30. (a) $\frac{20}{2}\{2(-7) + 19d\} = 620 \Leftrightarrow d = 4$

(b) $u_{78} = -7 + 77(4) = 301$

31. **METHOD 1**

substituting into formula for S_{40} $1900 = \frac{40(u_1 + 106)}{2} \Leftrightarrow u_1 = -11$

substituting into formula for u_{40} $106 = -11 + 39d \Leftrightarrow d = 3$

METHOD 2

substituting into formula for S_{40} $20(2u_1 + 39d) = 1900$

substituting into formula for u_{40} $106 = u_1 + 39d$

Solve the system $u_1 = -11$, $d = 3$

32. (a) $u_1 = 10 \quad u_2 = 15$ (b) $d = 5$ (c) $S_3 = 45 \quad S_4 = 70$ (d) $S_n = \frac{n}{2}(5n+15)$

33. (a) 23 (b) 99 (c) 1265

34. Arithmetic progression: 85, 78, 71, ...

$u_1 = 85$, $d = -7$

$u_n = 85 - 7(n-1) = 92 - 7n \quad u_n > 0 \Rightarrow n \le 13.$

$S_{13} = \frac{13}{2}(u_1 + u_{13}) = \frac{13}{2}(85 + 1) = 559$

35. $u_1 = -6$ and $d = 7$

$S_n = \frac{n}{2}(2u_1 + (n-1)d) \Rightarrow S_n = \frac{n}{2}(7n - 19)$

Solving $S_n > 10000 \Rightarrow n > 54.8$, The least number of terms is 55

36. (a) $(2a + 4) - (a + 3) = (a + 9) - (2a + 4) \Leftrightarrow 2a = 4 \Leftrightarrow a = 2$

(b) the terms are 5, 9, 11 which are indeed in AS with $d = 2$

PROBLEMS

37. Arithmetic sequence $d = 3$

$n = 1250 \qquad S = \frac{1250}{2}(3 + 3750) = 2\,345\,625$

38. Arithmetic sequence $u_1 = 200 \quad d = 30$

(a) Distance in final week = $200 + 51 \times 30 = 1730$ m

(b) Total distance = $\frac{52}{2}[2.200 + 51.30] = 50180$m

39. (a) AS with $u_1 = 15 \quad d = 2 \quad n = 20$

$u_{20} = 15 + (20 - 1) \times 2 = 53$ (that is, 53 seats in the 20th row)

(b) $S_{20} = \frac{20}{2}(15 + 53) = 680$ (that is, 680 seats in total)

40. (a) $u_1 = 1000 \Leftrightarrow u_n = 1000 + (n - 1)250 = 10\,000 \Leftrightarrow n = 37$. She runs 10 km on the 37th day.

(b) $S_{37} = \frac{37}{2}(1000 + 10\,000)$ She has run a total of 203.5 km

41. $81 = \frac{n}{2}(1.5 + 7.5) \Rightarrow n = 18$

$1.5 + 17d = 7.5 \Rightarrow d = \frac{6}{17}$

42. (a) $T_1 = 100 \quad d = 25$

$T_{17} = 100 + (17 - 1) \times 25 = \$500$

(b) $S_{17} = \frac{17}{2}(100 + 500) = \$5100$

43. (a) $45000 + 4 \times 1750 = 52000$ USD

(b) $\frac{10}{2}(2(45000) + 9 \times (1750)) = 528750$ USD (Accept 529000)

44. (a) $20 = u_1 + 3d$

$32 = u_1 + 7d$

$d = 3$ (and $u_1 = 11$)

(b) $\frac{10}{2}(2 \times 11 + 9 \times 3) = 245$

45. 4^{th} term $= a + 3d$

8^{th} term $= a + 7d$

20^{th} term $= a + 19d$

We have two relations:

$a + 7d = 2(a + 3d)$

$a + 19d = 4000$

The solution is $d = 200$ (and $a = 200$)

46. (a) $u_{21} = 24 + 20 \times (16) = 344$

(b) $S_{31} = \frac{31}{2}[2(24) + (31 - 1)(16)] = 8184$

B. Paper 2 questions (LONG)

47. (a) $u_1 = 1$, $n = 20$, $u_{20} = 20$ ($u_1 = 1$, $n = 20$, $d = 1$)

$S_{20} = \dfrac{(1+20)20}{2}$ (or $S = \dfrac{20}{2}(2\times1+19\times1)$) $= 210$

(b) Let there be n cans in bottom row

$S_n = 3240 \Leftrightarrow \dfrac{(1+n)n}{2} = 3240 \Leftrightarrow n^2 + n - 6480 = 0 \Leftrightarrow n = 80$ or $n = -81$

So $n = 80$

(c) (i) $S = \dfrac{(1+n)n}{2} \Leftrightarrow 2S = n^2 + n \Leftrightarrow n^2 + n - 2S = 0$

(ii) **METHOD 1**

Substituting $S = 2100$: $n^2 + n - 4200 = 0 \Leftrightarrow n = 64.3$, $n = -65.3$

n must be a (positive) integer, this equation does not have integer solutions.

METHOD 2

Trial and error: $S_{64} = 2080$, $S_{65} = 2145$ integer not possible here

48. (a) $u_1 = 1$, $d = 3$ $u_{11} = 31$

(b) $S_n = \dfrac{n}{2}(2 + (n-1)\times 3) = \dfrac{n}{2}(2 + 3n - 3) = \dfrac{n}{2}(3n - 1)$.

(c) (i) $\dfrac{100}{2}(3 \times 100 - 1) = 14950$

(d) (i) $\dfrac{n}{2}(3n - 1) = 477 \Leftrightarrow 3n^2 - n = 954 \Leftrightarrow 3n^2 - n - 954 = 0$

(ii) 18

49. (a) $(5k - 2) - (2k + 3) = (10k - 15) - (5k - 2)$

$\Leftrightarrow 5k - 2 - 2k - 3 = 10k - 15 - 5k + 2$

$\Leftrightarrow 3k - 5 = 5k - 13 \Leftrightarrow -2k = -8$ or $2k = 8$

$\Leftrightarrow k = 4$

OR

$(2k + 3 + 10k - 15) \div 2 = 5k - 2$

$\Leftrightarrow 2k + 3 + 10k - 15 = 10k - 4$

$\Leftrightarrow k = 4$

(b) 11, 18, 25 (A1)

(c) 7 (A1)

(d) $U_{20} = 11 + 19 \times 7 = 144$

(e) $S_{15} = \dfrac{15}{2}(2 \times 11 + 14 \times 7) = 900$

50. (a) (i) 6, 9, 12

(ii) 3

(b) $x^2 - (x^2 - 3) = 4x - x^2 \Leftrightarrow x^2 - 4x^2 + 3 = 0 \Leftrightarrow x = 1$ or $x = 3$

So the other value is 1.

(d) (i) -2, 1, 4

(ii) 3

(iii) -2 + 1 + 4 + 7 = 10